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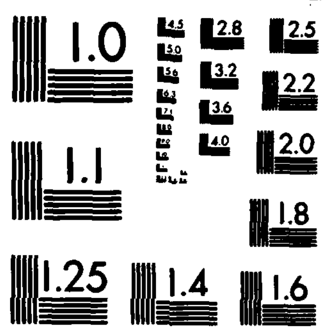

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SCALAR MODAL CONTROL OF A NONLINEAR

SYSTEM USING TIME VARYING GAIN

THESIS

Rosario Nici
Captain, USAF

AFIT/GA/AA/84D-8

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GA/AA/84D-8

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Approved for Public release; distribution unlimited

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USING TIME VARYING GAIN

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

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Rosario Nici, B.S., M.S.
Captain, USAF

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Preface

I want to thank Dr. William Wiesel for his guidance, insight, and general support, without which this thesis would not have been completed.

I also want to express my gratitude to my wife Jane and daughter Lindsay for their understanding during the past 18 months. I also want to thank Major David Olkowski and Capt John Ward for their help during times of need and assistance in confusing moments. As a last note, the entire GA 84D section deserves credit for the mutual help given during this masters program.

Table of Contents

	Page
Preface	ii
List of Figures	iv
List of Tables	v
List of Symbols	vi
Abstract	vii
I. Introduction	1
Area of Concern	2
Problem	3
Objective	3
II. Theory	4
Henon and Heiles Model	5
Floquet Theory	6
Controller	10
III. Results	17
Testing	29
IV. Conclusions	39
Recommendations	42
Bibliography	43
Vita	44

List of Figures

Figure		Page
1.	Reference Orbit - state variable X	18
2.	Reference Orbit - state variable Px	19
3.	Reference Orbit - state variable Y	20
4.	Reference Orbit - state variable Py	21
5.	Controllability Scalar for Unstable mode $g_i(t)$	25
6.	Gain k(t) - Fourier series Method	26
7.	Gain k(t) - Inverse $g_i(t)$ Method	28
8.	X state deviations Case 1, Zero Displacement .	32
9.	X state deviations Case 5, Zero Displacement .	33
10.	Gain k(t) - Inverse $g_i(t)$ Method, Case 8 . . .	36
11.	X state deviations Case 8, Zero Displacement .	37
12.	X state deviations Case 8, Zero Displacement Extended integration	38

List of Tables

Table Page

1. Orbit Values 17

2. $g_i(t)$ Fourier series 23

3. Testing Case Data 30

List of Symbols

$A(t)$	variational equation coefficient matrix
$B(t)$	control distribution matrix
f_i	i th row of the $F(t)$ matrix
$f(\bar{x})$	functional notation - nonlinear EOM
E	Henon and Heiles energy function
$F(t)$	matrix of eigenvectors of monodromy matrix
$g_i(t)$	controllability matrix
H	system Hamiltonian
I	identity matrix
J	Jordan form matrix of Poincare exponents
$k(t)$	time periodic controller gain
N	modal variable - c subscript for controlled
Px, Py	generalized momenta
t	independent variable time
t_o	initial time
T	period of reference orbit
$u(t)$	periodic control vector
x, y	position coordinates
$\bar{x}$	state variable
$\bar{x}_o$	reference state
$\Delta\bar{x}$	displacement - variation
$\phi(t, t_o)$	state transition matrix
$\phi(T, 0)$	monodromy matrix
w_i	Poincare exponent
w_i'	modal exponent

ABSTACT

A nonlinear system exhibited some relationship between system stability and controller off-time. An increase in the off-time of the controller from 6.5 % to 78.0 % resulted in the nonlinear system maintaining stability over 50 time periods.

A nonlinear system was controlled using feedback based on the unstable system parameters. The Henon and Heiles model was used to provide an unstable periodic orbit. The linearized equations of motion, using Floquet Theory, were developed into variational equations. A controller was derived based on a system characteristic: the controllability matrix. The controller gain was also calculated based on the controllabilty matrix.

Together with the nonlinear system the controller was integrated for many time periods using zero initial displacement. Stability was achieved for approximately 50 time periods of the system around the zero displacement conditions. Other areas were investigated, along with several other controller gains, without any stability noted.

SCALAR MODAL CONTROL OF A NONLINEAR SYSTEM USING TIME VARYING GAIN

CHAPTER 1

INTRODUCTION

Control of systems that are linear, especially constant coefficient systems, have been explored in great detail. This vast knowledge forms a basis upon which one can build and expand. The expansion into time periodic linear systems is another logical step. Floquet Theory provides the tools and analytical development needed to deal with this next step(4:55). The nonlinear system is an extension of the linear system and the utility of a linear system model relies on the validity of the assumption that the model is a close approximation of the nonlinear system. Linearizing a nonlinear system is done for many reasons: 1. ease of understanding the system, 2. reduced complexity in dealing with the system, 3. tools are readily available for investigation, and 4. from previous experience it is valid (i.e. it works).

One can take a nonlinear system, linearize it (details later), and try to control it. The question, "Whether or

not the control worked?" is closely tied to the stability information of the system. Does the desired action take place even if the setup wasn't perfect? Or in perturbation theory, " Does the displacement from a reference state disappear as time goes on?" These are good questions, but the overall test, the test that indicates success or failure, is based on the reactions of the nonlinear system under the same circumstances. Stability is the grading criteria for the control function. Stability is defined as having a tendency to return to a desired state. A system with neutral stability doesn't exhibit any tendencies to return to a desired state, while an unstable system when disturbed gets further and further from the desired state.

AREA OF CONCERN

With the background established, focusing can begin. Several individuals have worked in the area of cotrolling nonlinear systems with time periodic coefficients under the direction of Dr. Calico and Dr. Wiesel at the Air Force Institute of Technology. The specifics of this report deal with the work started by Captain Liby in his Masters Thesis in 1983(7:1). Captain Liby investigated a scalar modal controller and how changing the gain affects the region of stability. In this report an effort was made to understand and control a nonlinear time periodic system and make observations concerning this system.

PROBLEM

A nonlinear system will be chosen as a medium through which observations can be made. It will be linearized and a reference state established for investigation using perturbation theory. The reference state must provide a single unstable mode upon which scalar control using time varying gain can be tested. Once the linear model is stable, the nonlinear system with a controller added will be checked for stability.

OBJECTIVE

The objective therefore is to choose a gain which accomplishes two sub-goals: 1. ensure stability in the linear system, and 2. stabilizes the nonlinear system even after many time periods. (Note: the second objective requires the full nonlinear system plus the controller to be numerically integrated over several periods.)

CHAPTER 2

THEORY

The continuation of research in scalar modal control relating the linear model with the nonlinear system is the main focus of this report. Captain Liby's thesis worked on a scalar model controller of an unstable system (i.e. the system had an unstable Poincare exponent). After integrating the nonlinear system with the linear controller Captain Liby's system was diverging/unstable. Using the linear system, the controller moved the positive Poincare exponent into the stable region, however the nonlinear system did not exhibit the same tendencies. The thrust of my work will be to determine a gain function such that the nonlinear system is also stable after many time periods.

A linear model was developed based on the nonlinear unstable periodic orbit used in Captain Liby's thesis. The Henon and Heiles development was the basis of this orbit. Floquet theory was used to change from state to modal variables and scalar feedback control was used to control the single unstable mode. Relationships relating the nonlinear equations to the gain used for the model feedback were explored. Time varying gain was used and the difficulties in finding the gain were examined.

HENON and HEILES MODEL

In order to determine the gain and solve the instability problem, a test case must be chosen. The Henon and Heiles model was used for the very reason that it was first developed because it provides enough complexity to generate nontrivial solutions yet still maintain simplicity for the ease of understanding relationships(5:75). The potential function and total energy in this model are:

$$U(x,y) = 1/2(x^2+y^2+2x^2y-2/3y^3) \quad (1)$$

$$E = U(x,y) + 1/2(\ (dx/dt)^2 + (dy/dt)^2\ ) \quad (2)$$

Changing to canonical units where Px and Py are the generalized momenta of x and y respectively, the Hamiltonian is

$$H = 1/2(x^2+ y^2 + 2x^2y - 2/3y^3) + 1/2(Px^2 + Py^2) \quad (3)$$

The equations of motion are

$$dx/dt = d\ H\ /\ dPx = Px \quad (4)$$

$$dPx/dt = -\ d\ H\ /\ dx = -x\ -2xy \quad (5)$$

$$dy/dt = d\ H\ /\ dPy = Py \quad (6)$$

$$dPy/dt = -\ d\ H\ /\ dy = -y\ -x^2 + y^2 \quad (7)$$

Since there is no time dependence in the Hamiltonian then the total energy (2) is a constant.

The energy may take on any value. Each value of the energy equates to a different surface on which the state variables vary. But once the energy value of the surface is set, it remains a constant.

The state variables used are the position and their momenta respectively.

$$\hat{X} = [\ x\ P_x\ y\ P_y\ ]^T \tag{8}$$

The equations of motion are nonlinear and can be represented in functional notation as

$$d\hat{X}/dt = f(\hat{X}) \tag{9}$$

The time rate of change of the state is defined by the function f. When using a controller, a reference state is also required to compare the current state to the desired performance. For analytical developments, a linear set of variational equations were developed. The assumption of a small displacement from a reference state must hold for the linear system (equations of variation) to be a valid representation of the nonlinear system(11:24).

FLOQUET THEORY

An unstable periodic orbit of period T was used as a reference state $\hat{X}_o$. A small displacement $\Delta\hat{X}$ was used to expand the f function in a Taylor's series about the reference state,

$$d(\hat{x}_o + \Delta\hat{x})/dt = f(\hat{x}_o) + \Delta\hat{x}\, df/d\hat{x}|_{\hat{x}=\hat{x}_o} + O(2) \qquad (10)$$

where O(2) are terms of order 2 and above in $\Delta\hat{x}$.
Using equation (9) evaluated at $\hat{x}_o$, equation (10) in matrix form is:

$$d\Delta\hat{x}/dt = A(t)\Delta\hat{x} \qquad (11)$$

where $$A(t) = df/dt|_{\hat{x}=\hat{x}_o} \qquad (12)$$

The elements of A, A_{ij} are the partial derivatives of the ith equation of motion with respect to the jth element of the state vector. Equation (11) is a linear time-periodic differential equation, since A(t) has period T. The state transition matrix relates the state at time equal to zero to any time:

$$\hat{x}(t) = \varphi(t,0)\hat{x}(0) \qquad (13)$$

The state transition matrix is not periodic, but at t = T, $\varphi(T,0)$ is called the monodromy matrix. The matrix $\varphi(t,0)$ also satisfies the differential equation (11):

$$d\varphi(t,0)/dt = A(t)\varphi(t,0) \qquad (14)$$

At time equals zero, examining equation (13) reveals that $\varphi(0,0)$ is the identity matrix. Thus the state transition matrix can be determined by solving equation (14) with the identity matrix as an initial condition.

The main result of Floquet is $\varphi(t,0)$ can be factored using two matricies, F and J:

$$\varphi(t,0) = F(t)e^{Jt}F^{-1}(0) \tag{15}$$

The matrix F(t) is periodic with the same period T as the original system. The J matrix is a constant matrix which when diagonalized has diagonal entries termed the Poincare exponents(2:3). Complex exponents can be represented as two real elements in Jordan Normal Block form. With t=T and remembering the periodicity of the F matrix, $\varphi(T,0)$ can be calculated below

$$\varphi(T,0) = F(0)e^{JT}F^{-1}(0) \tag{16}$$

where F(0) is the matrix of eigenvectors of $\varphi(T,0)$ (2:5). The eigenvalues of the monodromy matrix are related to the Poincare exponents (w_i) by

$$\lambda_i = e^{(w_i T)} \tag{17}$$

Now F(t) can be found by substituting equation (15) into equation (11) for the state transition matrix shown below.

$$d\varphi(t,0)/dt = d(F(t)e^{Jt}F^{-1}(0))/dt \tag{18}$$

$$= dF(t)/dt[e^{Jt}F^{-1}(0)]+[F(t)d(e^{Jt})/dtF^{-1}(0)] \tag{19}$$

Since $F^{-1}(0)$ is a constant, $dF^{-1}(0)/dt$ is zero. With some rearranging of terms, the time derivative of F(t) can be found:

$$A(t)F(t)e^{Jt}F^{-1}(0) = dF(t)/dt\,e^{Jt}F^{-1}(0)+JF(t)e^{Jt}F^{-1}(0) \quad (20)$$

After postmultiplying by $F(0)e^{-Jt}$, equation (20) becomes

$$A(t)F(t) = dF(t)/dt + JF(t) \quad (21)$$

or

$$dF(t)/dt = A(t)F(t) - F(t)J \quad (22)$$

The inverse of F(t) can also be found by using the identity relationship $FF^{-1} = I$ which with some development becomes

$$dF^{-1}(t)/dt = -F^{-1}(t)A(t) + JF^{-1}(t) \quad (23)$$

The background is established to introduce a change of variables to simplify the linear model:

$$\hat{x}(t) = F(t)N(t) \quad (24)$$

substituting into equation (11)

$$dF(t)N(t)/dt = N(t)dF(t)/dt + F(t)dN(t)/dt \quad (25)$$

using the latter relationship, and the right hand side of equation (11) yields

$$A(t)F(t)N(t) = N(t)dF(t)/dt + F(t)dN(t)/dt \quad (26)$$

which when substituting in equation (22) yields

$$= N(t)[A(t)F(t) - F(t)J] + F(t)dN(t)/dt \quad (27)$$

Rearranging and premultiplying by $F^{-1}(t)$ results in

$$dN(t)/dt = JN(t) \tag{28}$$

Thus the eigenvector matrix reduces the periodic system into a constant coefficient system(2:7). The constant coefficient system can be easily solved and the diagonal entries in the J matrix (Poincare exponents) indeed give the stability information of the system. A positive diagonal element shows instability, were a negative value shows stability. Since this study is only concerned with the scalar control of one mode, the case of only one mode with a positive value will be used. Feedback control is used, but the state variable $\underline{X}$, not the model variable N are used. The state variables are readily available and have physical representations (i.e. velocity).

CONTROLLER

Equation (11) is modified to include feedback control:

$$d\underline{X}/dt = A(t)\underline{X}(t) + B(t)u(t) \tag{29}$$

where B(t) determines the distribution of the control and can be a function of time with the same period T, $u(t) = G(t)\underline{X}(t)$ the feedback control, and G(t) the gain function. B(t) was chosen to affect only the momenta states. Physically this relates to changing the velocity, whereas the position states can not be changed instantaneously. Therefore B(t) is in the form

$$B = [\ 0 \quad 1 \quad 0 \quad 1\ ]^T \tag{30}$$

substituting equation (24) into (29) results in

$$d(F(t)N(t))/dt = A(t)F(t)N(t) + B(t)G(t)F(t)N(t) \tag{31}$$

$$dF(t)/dtN(t)+F(t)dN(t)/dt = [A(t)F(t)+B(t)G(t)F(t)]N(t) \tag{32}$$

rearranging and substituting for dF(t)/dt, equation (22)

$$dN(t)/dt = [J + F^{-1}(t)B(t)G(t)F(t)]N(t) \tag{33}$$

Let $g(t)=F^{-1}(t)B(t)$ be defined as the controllability matrix and $k(t) = G(t)F(t)$ be defined as the new gain matrix Only the unstable mode will be fedback. The controlled system equations take the form below, where N_c is the closed loop state.

$$dN_c/dt = \begin{bmatrix} w_1 & 0 & kg_1 & 0 \\ 0 & w_2 & kg_2 & 0 \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ 0 & 0 & kg_i+w_i & 0 \\ 0 & 0 & kg_n & w_n \end{bmatrix} N_c \tag{34}$$

Only the ith element of the closed loop system has a different Poincare exponent from the open loop system(2:9). The unstable mode equation (35) can be solved by the use of an integrating factor, yielding (36):

$$dN_{ci}/dt = [\ w_i + k(t)g_i(t)\ ]N_{ci} \tag{35}$$

$$N_{ci}(t)=N_{ci}(0)\exp[{}_0\!\int^t (w_i+k(t)g_i(t))dt] \tag{36}$$

Both $g_i(t)$ and $k(t)$ can be periodic and expressed as Fourier series. The multiplication can be expanded as a constant part and a periodic part. The state transition matrix is the exponential part of equation (36); when evaluated at the period T, the periodic part goes to zero (exp(0)=1) and stablility information is contained in the eigenvalue. Hence a new modal exponent is found.

$$w_i' = w_i + k_c g_{ic} \tag{37}$$

where w_i' is the new modal exponent, and c subscipt indicates the constant part of g_i and k.

Multiplying two Fourier series can be resolved into separate sine and cosine function and higher order harmonics. A constant part will be contributed from many products of higher harmonics, but can readily be reduced into sine and cosine components with the trigonometric sum and difference formulas.

This is one method for determining the gain, k(t). The merits of this method will be discussed later. Another method for determining the gain was developed because of two reasons. First, the difficulty in multiplying two Fourier series. Second, special care must be taken so the linear assumptions are not violated even though the linear system is stable. Capt Liby, in his thesis, chose a gain and stablized the linear system, however the nonlinear system was not stable in all cases over many time periods. Capt

Liby used a gain equal to a constant times the first harmonic sine term (7:16). By substituting this gain into equation (36), he found a new Poincare exponent through equation (37). However, even though the integral of the remaining periodic functions in equation (36) are evaluated over one period and hence zero, during the period the values can become very large. When the modal variable is large the variation becomes large (7:39). Hence, the small displacement assumption might be violated by the transient response of the linear system. Once the assumption of linearity is invalidated, the linear system is no longer a valid model of the nonlinear system.

The gain selected was developed by examining equation (36) for the modal variable as a function of time. The k(t) function was a discrete, evenly spaced inverse of the g_i function multiplied by the desired constant so that the sum of kg_i and w_i is now a negative (stable) number. Explicitly, this drives the entire $k(t)g_i(t)$ = constant. If this can be done successfully, there would not be any periodic function to cause the modal variable to become large during the period. The variation will not be large, and the assumptions of linearity will remain valid.The only discrepancy is the zero points of the g_i function. A maximum value of the k function is equivalent to limiting the control forces imposed on real systems. A zero value can be used where a higher value is required; this equates to a

shutoff of the control mechanism when the actual control forces required approach infinity.

The particular value of the constant $(k(t)g_i(t))$ depends on several factors. Certainly the amount or tendency of instability already present in the system will impose restrictions on the $k(t)g_i(t)$ value. Obviously, if during one time period there is not a tendency to return to the reference orbit, the deviation would grow in succeeding periods. This fact provides a starting point for using values of the $k(t)g_i(t)$, it must be at least as negative as the current w_i (Poincare exponent) is positive.

Another factor relates to the off-time of the control mechanism. When a zero point of $g_i(t)$ is reached during the orbit the controller is turned off. Otherwise an infinite control force would be required. The futility of using a finite control input when an infinite response is self-evident. Therefore, a trade-off is developed between the percentage of off-time and the amount of stability introduced by the controller (i.e. how big was the constant $k(t)g_i(t)$).

A final factor to be considered is whether the gain is realistic. A control system that works while requiring elaborate and extremely complex system operations may not be an overall improvement but a hinderence. This certainly is a lessor criterion but should be kept in mind when developing controllers.

Once the gain function is calculated the next step in the procedure is incorporating the gain in the nonlinear system. Refer back to equation (9). It now can be rewritten with a control function added.

$$d\check{X}/dt = f(\check{X}) + \check{C}ontrol \qquad (38)$$

where the $\check{C}$ontrol is a n x 1 matrix. The composition of the control function is determined by going through the transformation of (1) feeding back the state variables, (2) converting to modal variables, (3) applying scalar control of the unstable mode, and finally (4) multiplying by the gain function. This can be outlined by rearranging equation (24) in terms of displacement from a reference orbit.

$$N(t) = F^{-1}(t)\check{X}(t) \qquad (24)$$

$$R(t) = [k_1(t) \quad 0 \quad 0 \quad 0 \quad] \qquad (39)$$

where R(t) assures the gain of only the unstable mode. B(t), equation (30) is used to properly distribute the controller in the proper order,

$$B = [\quad 0 \quad 1 \quad 0 \quad 1 \quad]^T \qquad (30)$$

combining all terms,

$$\check{C}ontrol = B(t)R(t)F^{-1}(t)[\ x(t) - x_o(t)\ ] \qquad (40)$$

which reduces to:

$$\mathrm{\check{c}ontrol} = k_i[f_i^{-1}(t)\ ][\ x(t)-x_o(t)] \qquad (41)$$

where $f_i^{-1}(t)$ is the row of $F^{-1}(t)$ associated with the unstable mode, k_i is the gain, and $x(t)-x_o(t)$ is the displacement from the reference orbit at any time.

CHAPTER 3

RESULTS

Using the energy function (2) of the Henon and Heiles problem, an unstable periodic orbit was calculated. Initial conditions of energy= 0.125 , Py(0)=0, x(0)=0 were used with an estimate for y(0) and the period T. A predictor-corrector numerical integrator was used to match end points on the orbit. The period of the orbit and initial state values are found in table 1.

ORBIT VALUES

T	=	32.040441196
E	=	.124999316
X(0)	=	.000000134
PX(0)	=	.499704388
Y(0)	=	-.017054328
PY(0)	=	.000001441

TABLE 1.

The values for the reference orbit for each element in the state x, Px, y, Py are shown in Fig 1 - Fig 4 respectively. These figures show 50 points evenly spaced over the period. All other figures that are used to show stability information are deviations from these values throughout the period.

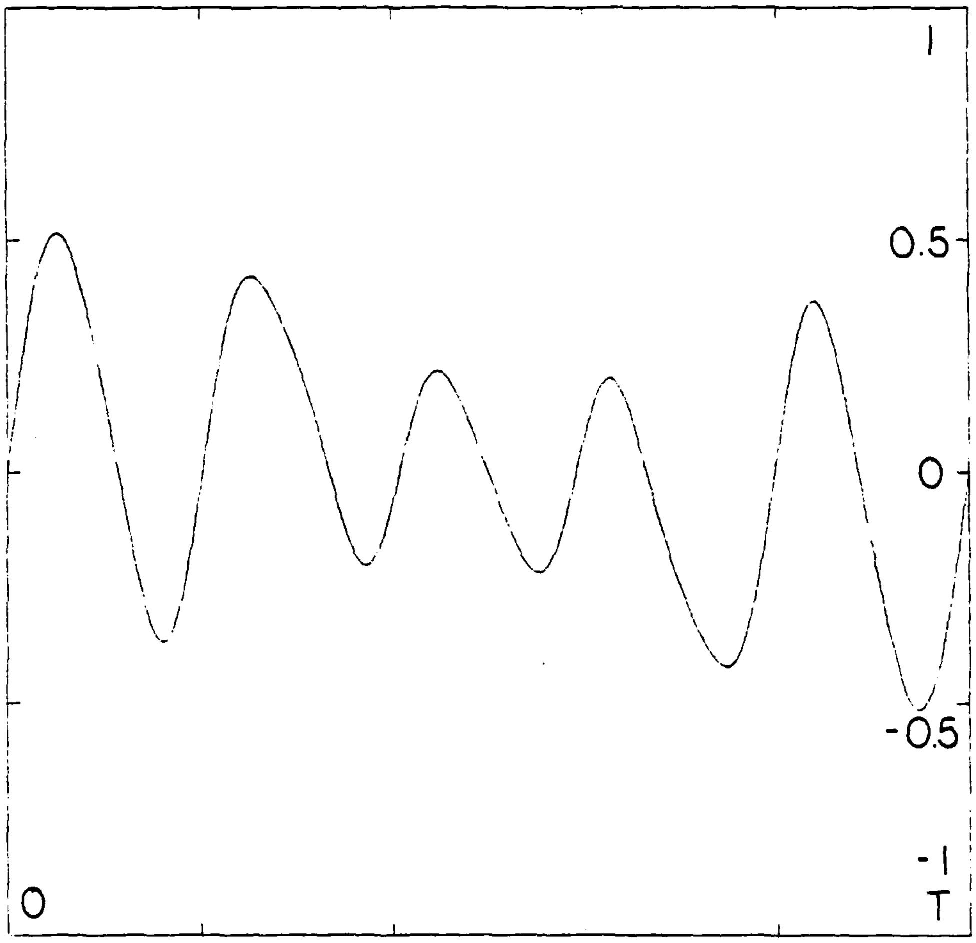

Fig 1. Reference Orbit - state variable X

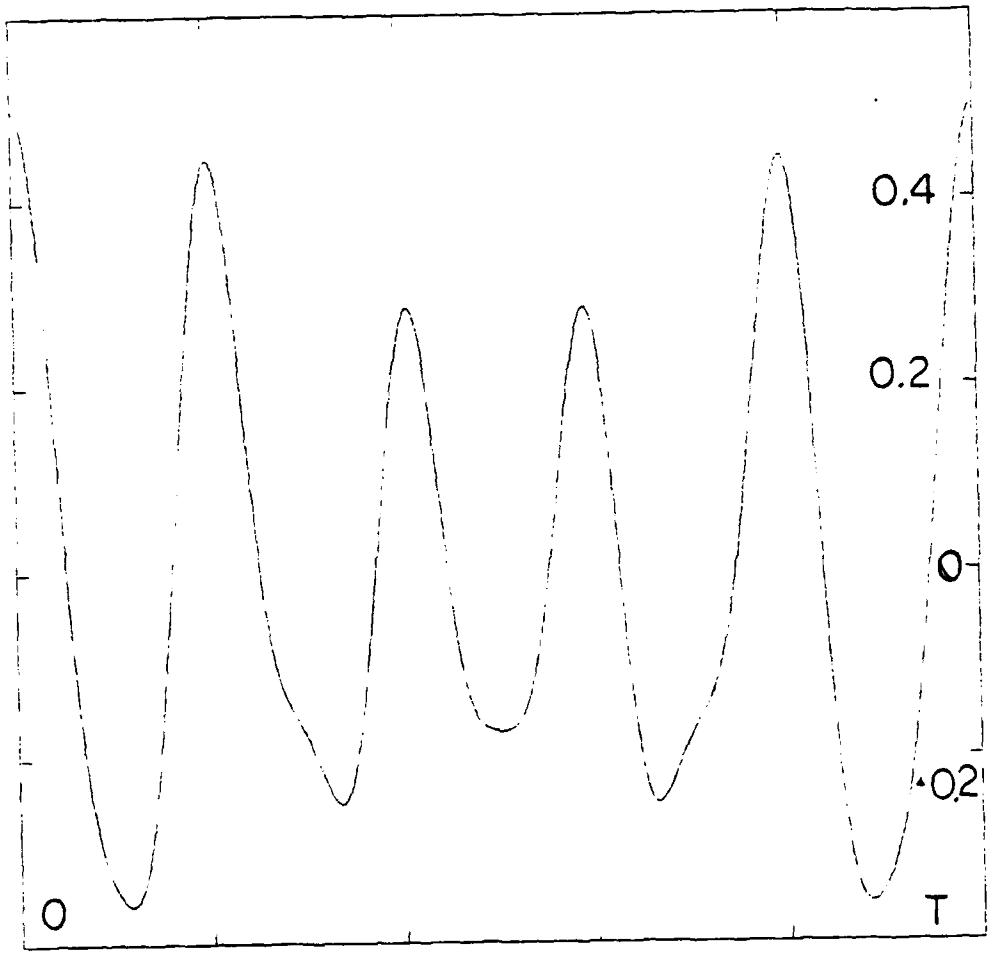

Fig 2. Reference Orbit - state variable Px

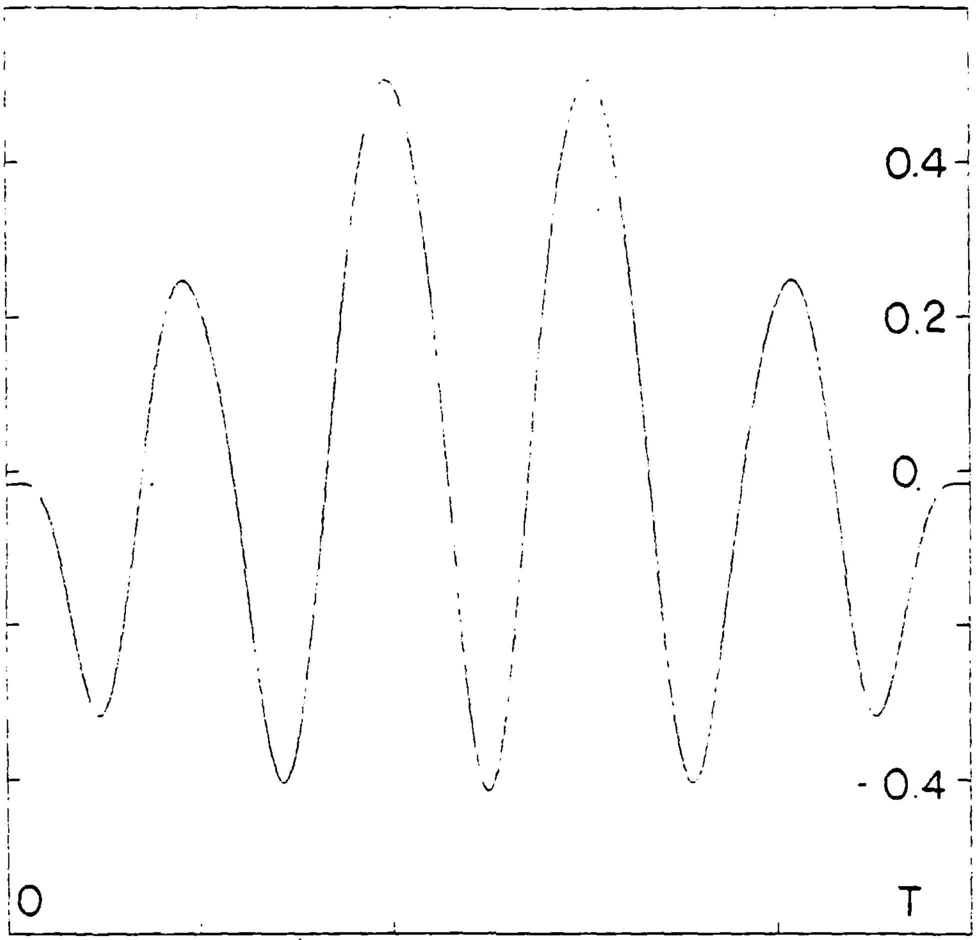

Fig 3. Reference Orbit - state variable Y

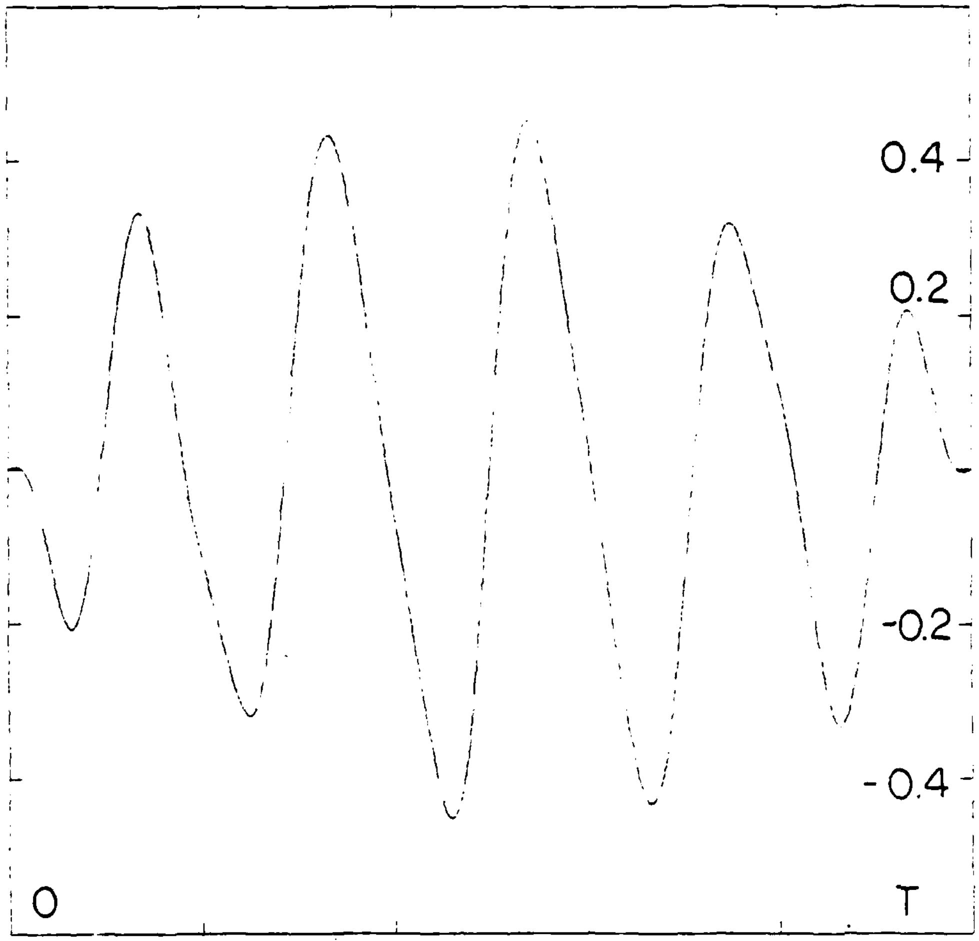

Fig 4. Reference Orbit - state variable Py

In conjunction with finding the periodic orbit, the state transition matrix (18) was calculated at each integration step and the monodromy matrix, with the associated stability information was available at t = T. The eigenvalues (17) are related to the Poincare exponents and revealed a single unstable mode. The eigenvector associated with the positive Poincare exponent is a column of the F(0) matrix. The F(0) matrix was inverted and the row equation (42) corresponding to the unstable eigenvalue was used as an initial condition for equation (23). The $F^{-1}(t)$ row was integrated for one period T. Since $F^{-1}(t)$ has the same period as the linear system, once it is calculated for one period it is known for all time. The row of $F^{-1}(0)$ is shown below where $f_i^{-1}(0)$ is the row of F^{-1} associated with w_i.

$$w_i = 0.083668212 \tag{42}$$

$$f_i^{-1}(0) = [.013240546 \;\; .824668173 \;\; .417510018 \;\; -.3816845651] \tag{43}$$

One must remember the controllability matrix g(t) is defined below:

$$g(t) = F^{-1}(t)B(t) \tag{44}$$

The controllability function matrix used in equation (35) reduces equation (36) to control of the unstable mode. The g_i in equation (36) can be found by

$$g_i(t) = f_i^{-1}(t)B(t) \tag{45}$$

where $g_i(t)$ is a 1 x 1 matrix, i.e. scalar modal control. Hence,

$$g_i(t) = f_{i2}^{-1}(t) + f_{i4}^{-1}(t) \tag{46}$$

The scalar $g_i(t)$ is the sum of the second and fourth element of the row of $F^{-1}(t)$ corresponding to the positive Poincare exponent.

In all of the above computations periodic functions were represented by Fourier series (1:109). The Fourier series for $g_i(t)$, first six terms, are shown in Table 2.

$g_i(t)$ Fourier series

	$\cos(nw_o t)$		$\sin(nw_o t)$
a_o	.0000077		
a_1	.3300262	b_1	.4913984
a_2	.1247512	b_2	-.1374558
a_3	.0000185	b_3	-.0000165
a_4	-.5316386	b_4	-.1825139
a_5	.3743597	b_5	-.4790797
a_6	-.0000052	b_6	-.0000148

TABLE 2.

Once the controllability matrix is calculated the linear system can be stabilized with a judicious choice of gain. Recall equation (36), and if the sum $w_i + k(t)g_i(t)$ is less than zero then the controlled modal variable $N_{ci}(t)$ displacement decays with time. This implies the system returns to the reference state; stability is achieved. For

this specific case w_i (42) is slightly positive. If k(t) and $g_i(t)$ are represented by fourier series, the multiplication of the two in general requires multiplying two infinite series. If n terms are used for each, k(t) theoretically could be calculated from 2n + 1 linear equations. There would be n cosine and sine equations and one constant part equation (37). The ideal restriction of a zero coefficient for the first n terms was explored. This would provide for the first n higher harmonics to be zero. Specifically, 28 terms for a total of 57 equations were used to determine a k(t). The new exponent w(t) was plotted and showed non-zero higher harmonics, high ocillations. The reason for the poor outcome is obvious when one rearranges the variables in exponent of equation (36).

$$k(t) = (w_i` - w_i)/ g_i(t) \qquad (47)$$

where w_i is the old Poincare exponent and $w_i`$ is the desired Poincare exponent (negative for stability). When a zero of $g_i(t)$ is reached k(t) is undefined. A representation of k(t) shows this descrepency as g(t) approaches zero from plus to minus, k(t) changes from a large negative value to a large positive value over a short time change. The graph of $g_i(t)$ is shown in Fig. 5 and the k(t) determined in Fig. 6. If one looks closely the spikes on the k(t) graph correspond to the zeroes on the $g_i(t)$ graph.

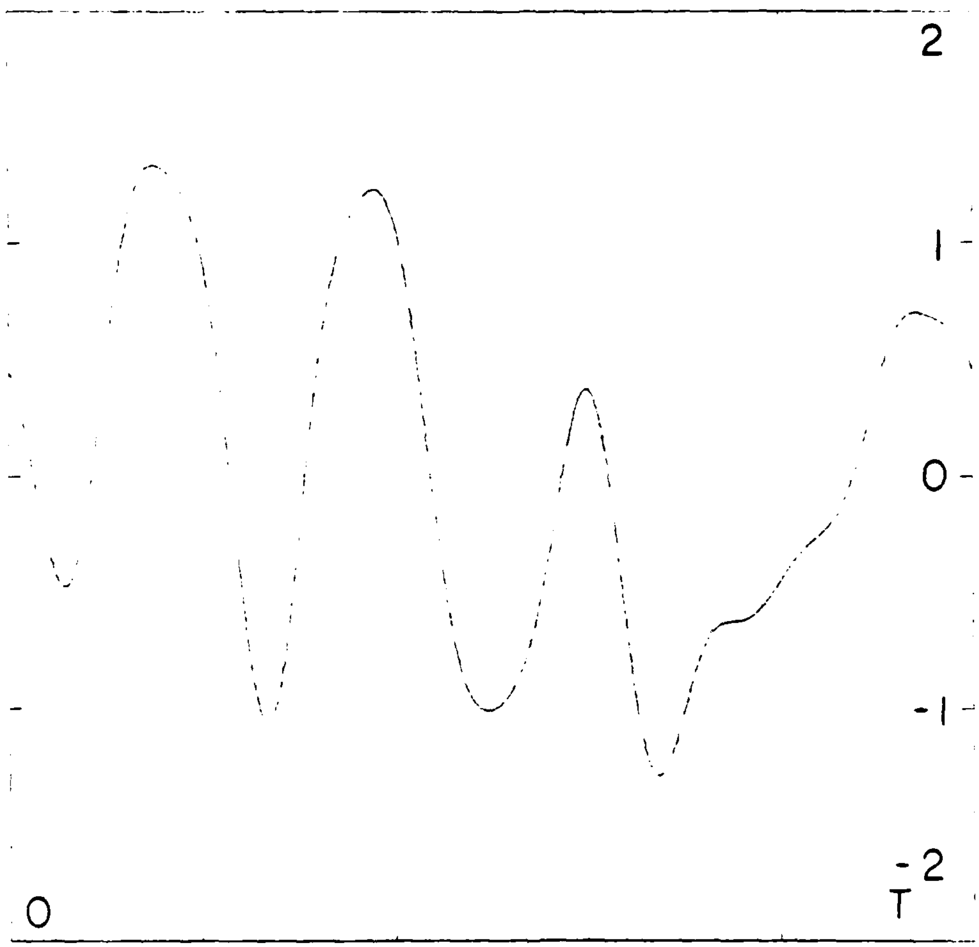

Fig 5. $g_1(t)$ - Controllability scalar for unstable mode.

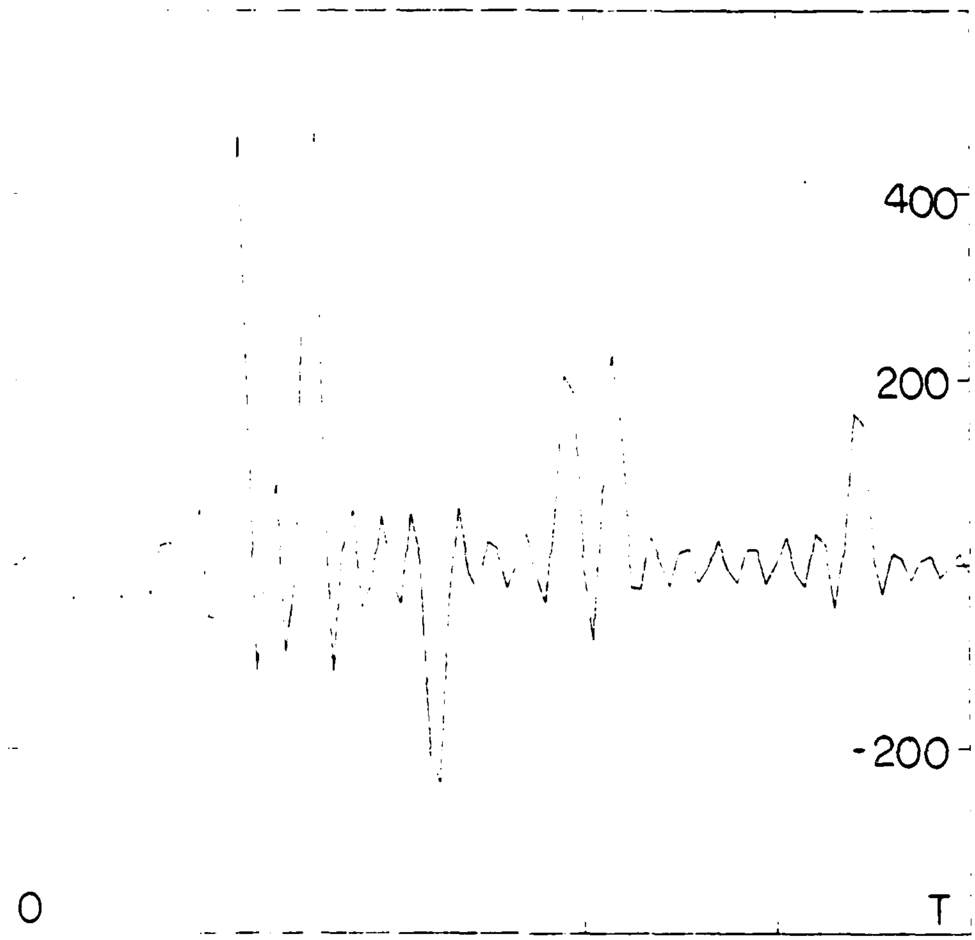

Fig 6. Gain k(t) - Fourier series Method

An attempt was made to eliminate all of the zeroes of $g_i(t)$, however, this is impossible because of the composition of $g_i(t)$. Since $g_i(t)$ is a linear combination of $f_2^{-1}(t)$ and $f_4^{-1}(t)$ one could determine a ratio of the two to limit the zeroes to only two. But the limit is two because the functions sine and cosine, have two zeroes for each period. With this in mind, $g_i(t)$ was examined and two restrictions to finding $k(t)$ were established: (1) a maximum value of 10 units was set for $k(t)$, and (2) when a higher value of $k(t)$ is determined from equation (47) then $k(t)$ was set to zero. This is just a simple restriction so an infinite input is not required from the controller. Following these restrictions and using 1000 discrete evenly spaced values between zero and the period, $k(t)$ is shown in Fig. 7.

The modal exponent, from equation (36), is now reduced to a bi-value function.

$$w_i' = w_i \qquad \text{unstable, } k = \text{zero} \qquad (48)$$

$$w_i' = w_i - kg_i \qquad \text{stable, } k \text{ non-zero} \qquad (49)$$

The percentage of the time out of one period when k is zero is inversely proportional to the maximum value of $k(t)$. The desired value, w_i' can compensate for this off-time. The restrictions imposed on the gain function simplify the equation (36) for the displacement of the modal control variable.

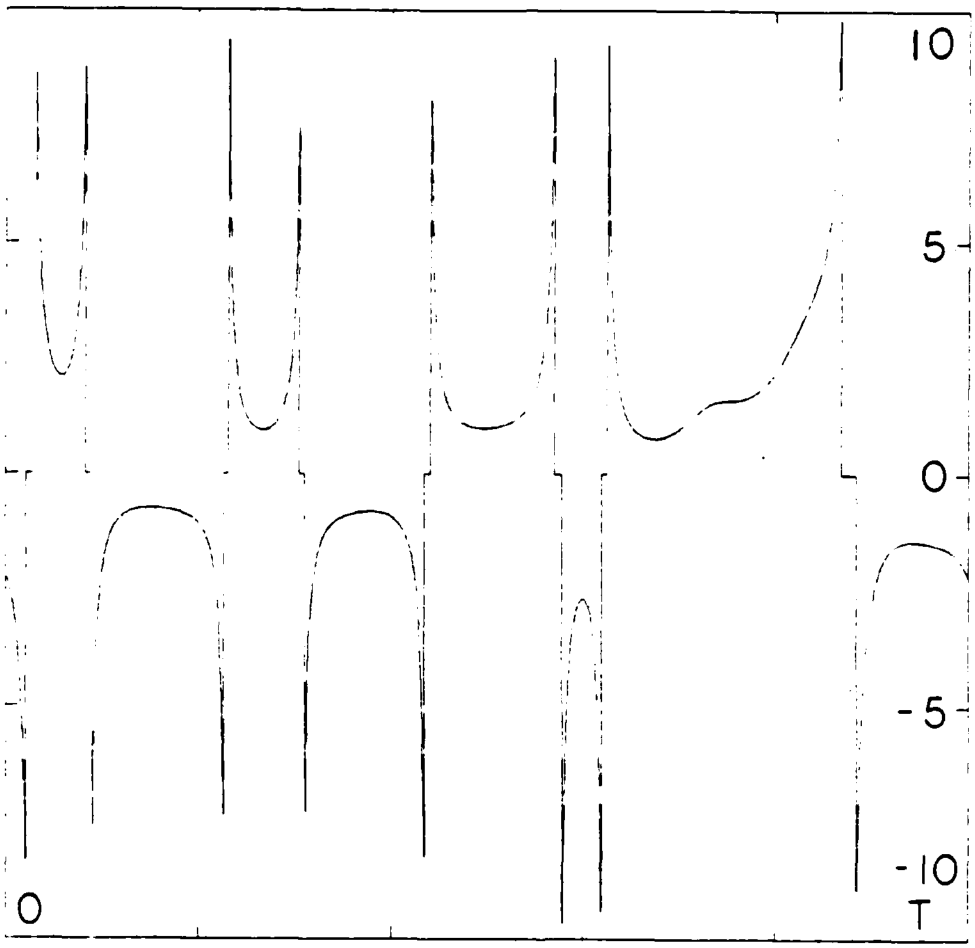

Fig 7. Gain k(t) - Inverse $g_i(t)$ Method

It now is either equation (50) or (51).

$$N_{ci}(t) = N_{ci}(0)e^{w_i' t} \tag{50}$$

$$N_{ci}(t) = N_{ci}(0)e^{w_i t} \qquad ,k = 0 \tag{51}$$

From equation (50) and (51) one can clearly show that the displacement is either decaying or slightly growing. A tradeoff exists between the amount of decay and growth based on relative magnitude of w_i' and w_i, which most of the time decays. The choice of gain may seem complicated but the goal is to stabilize the nonlinear system as well as the linear solution.

TESTING

Eight different gain configurations were tested and are shown in Table 3. The nonlinear system was numerically integrated for several periods, each period consisting of 1000 evenly spaced time steps. Prior to the first timestep the displacement of the current state from the reference state is recorded. Thereafter for each group of 20 timesteps for a total of 50 deviations are recorded for every period integrated. For example, if the nonlinear system is integrated over 18 periods and 50 deviations are recorded per period then there will be a total of 900 state displacements. These 900 points can be plotted for each variable of the state to provide a graphic representation of

the trend. Patterns can easily be seen and the controller affectiveness can also be evaluated.

TESTING CASE DATA

CASE	k(t)max	MAGNITUDE g(t) CUTOFF	k(t)g(t) (constant)	MODAL EXPONENTS off	MODAL EXPONENTS on
1	0	--	0	+.08367	--
2	10	0.02641	- .2641	+.08367	- .18045
3	11	0.03601	- .3961	+.08367	- .31252
4	10	0.05682	- .5682	+.08367	- .44458
5	1	0.10000	- .1000	+.08367	- .01633
6	10	0.10000	-1.0000	+.08367	- .91633
7	10	0.20000	-2.0000	+.08367	-1.91633
8	2	1.00000	-2.0000	+.08367	-1.91633

TABLE 3.

Each case is comprised of four main components: k(t)max, g(t) cutoff, k(t)g(t) constant , and modal exponent. The k(t)max is the maximum magnitude of the gain function. An arbitrary cutoff of around 10 was used to simulate an acceleration limitation.

The g(t) cutoff determines the on/off-time. During the gain function calculation in equation (47) when the magnitude of g(t) falls below the g(t) cutoff value, the k(t) gain function is assigned the value of zero. The lower the g(t) cutoff the less off-time. Case 6 with a g(t) cutoff of 0.1 has only 6.5% off-time. Whereas case 8, g(t) cutoff of 1.0, has an off-time of 78.0%

The constant value , k(t)g(t), determines the tendency

for decay of the displacement of the modal variables. (see equations (49) and (50)) The constant value can be used to compensate for increased off-time.

The modal exponent, as defined by equations (48) and (49), correspond to the off and on time of the controller. The modal exponent values range from a low of -.016 to a high of -1.9 in terms stability when the controller is on.

Case 1 is with zero gain, so the deviation of the nonlinear system to zero initial displacement from the reference state can be examined. The numerical roundoff and approximations excite the unstable mode. Fig 8. shows the deviations over 27 time periods of the first state variable. All four state variables exhibited the same characteristics so only the X state variable deviation is shown.

Cases 2-4 were used to compare results obtained by Capt Liby using a different controller. One can not compare results directly since Capt Liby used a sinusoidal controller and all above cases tested have on/off characteristics. Capt Liby found areas of stability and these were chosen as starting points. Several other points in the state space including zero displacement from the reference orbit were investigated. Deviations on the order of 10^{-1} were produced and no stable areas could be found.

Cases 5-6 were chosen because Capt Liby could not find any stable areas with comparable modal exponents. However, although similar deviations were produced as above, no

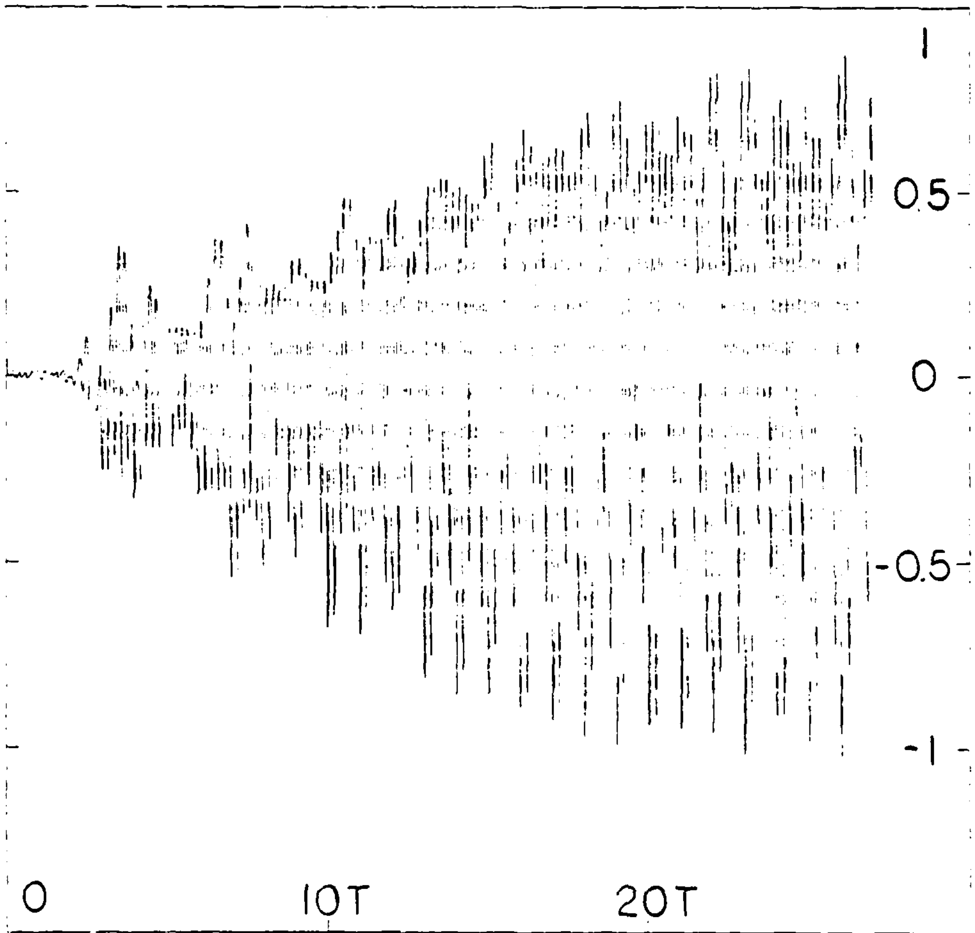

Fig 8. X state deviation Case 1, Zero Displacement

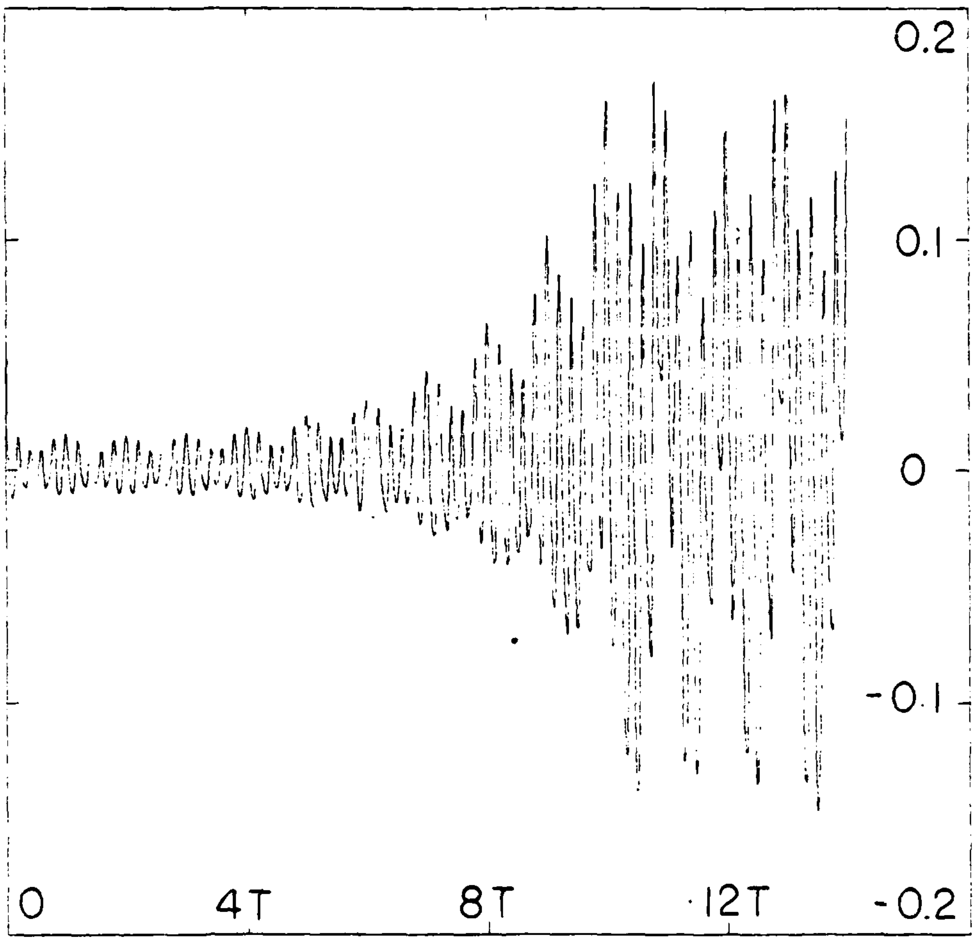

Fig 9. X state deviation Case 5, Zero Displacement

stable areas could be found. Fig 9. shows a typical trend of cases 2-6. Again only the X state variable is shown. Specifically, Fig 9. is case 5 integrated for 14 time periods with no initial displacement from the reference state.

Case 7 and 8 are an offshoot of the previous cases. The item that differentiates these case from the others is the amount of off-time. The highest off-time in the previous cases was case 6 with 6.5% off-time. Case 8 has an off-time of 78.0%, or only 22.0% on time. Both cases have the same k(t)g(t) constant, but differ in the g cutoff.

Case 7 is more similar to the previous cases. Several areas were investigated including zero initial displacement from the reference state and no areas of stability were found.

Case 8 was more promising. Fig 10. portrays the gain function k(t) for case 8. When compared to Fig 7. the increase in off-time is dramatic. Several areas were investigated and stability was found. Fig 11. shows zero initial displacement integrated for 27 time periods with decreasing deviations on the order 10^{-2}. Again only the X state deviations are shown as the other state variable characteristics are similar. One can compare Fig 11. to Fig 8. for evaluating the case 8 controller effectiveness. (note different scale) Fig 8. scale is 67 times bigger than Fig 11.

In Fig 11. the deviations do not disappear after tapering off. To ensure the stability of case 8, the zero initial conditions were integrated for 77 time periods, shown in Fig 12. After roughly 50 time periods with the deviations on the order of 10^{-2} the deviations suddenly grow and the system exhibits unstable behavior. This unstable behavior is assumed to be a numerical mismatching due to a possible synchronization problem between the actual period of the nonlinear system and the estimated period used in the integration. With an incorrect period the controller would not function at the proper time causing increased displacements instead of reduced displacements. However, the overall significance of Fig 12. is the depiciton of the stable deviations over many time periods (roughly 50).

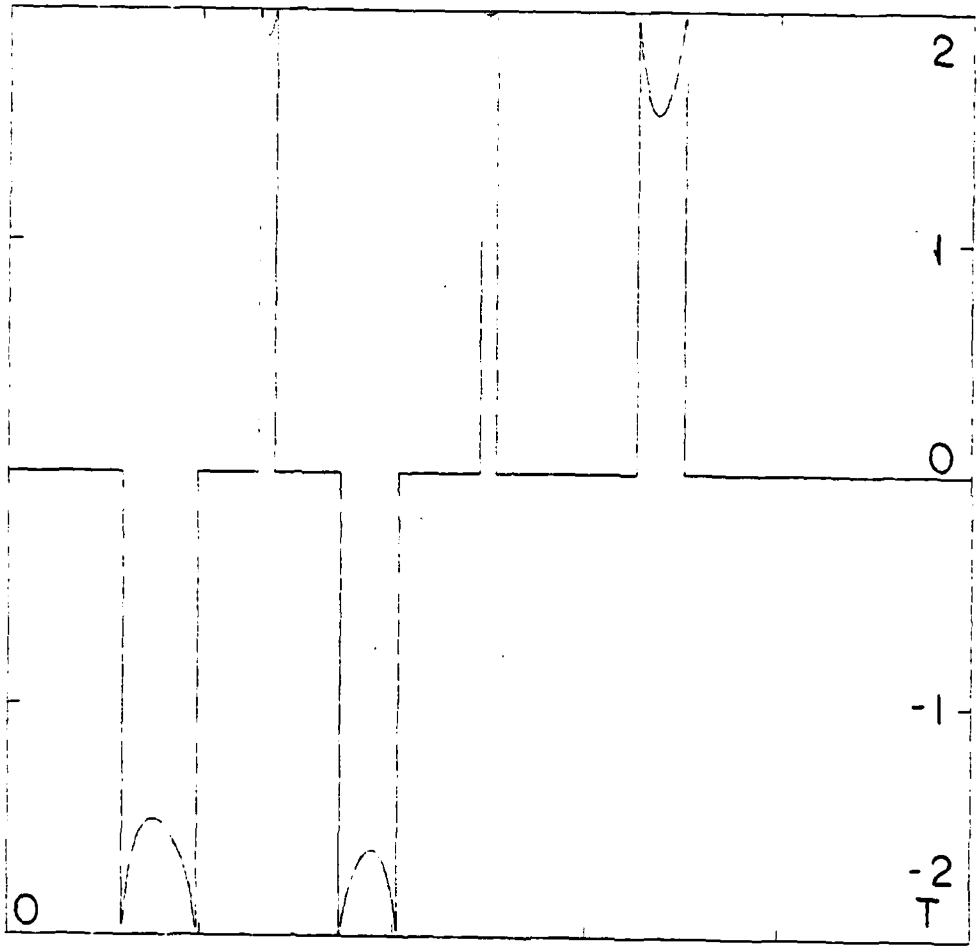

Fig 10. Gain k(t) - Inverse g(t) Method - Case 8

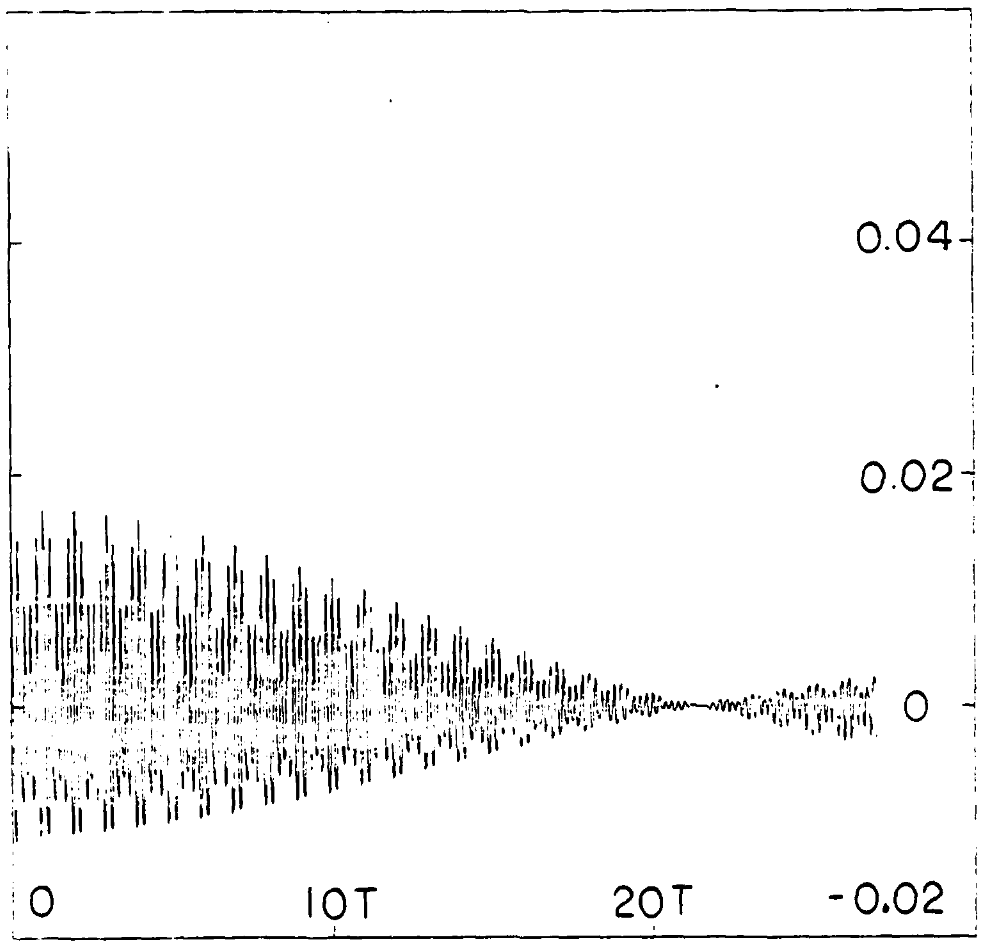

Fig 11. X state deviation Case 8, Zero Displacement

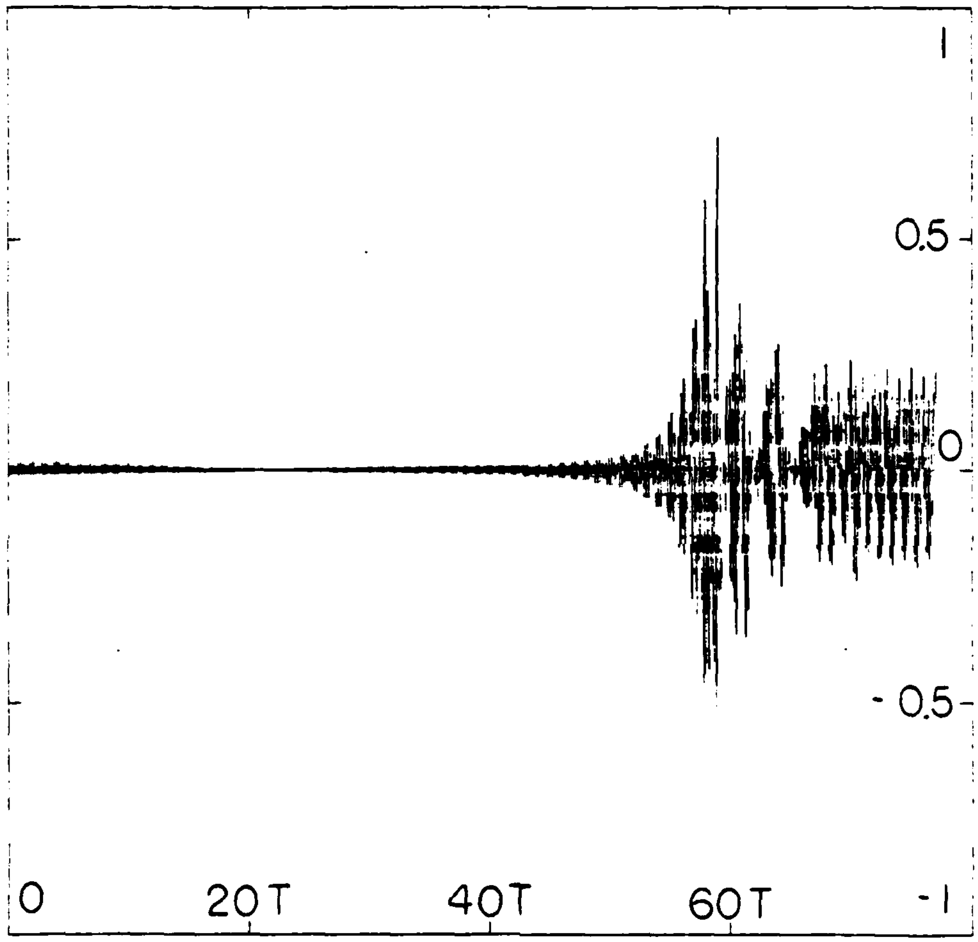

Fig 12. X state deviation Case 8, Zero Displacement Extended integration

CHAPTER 4

CONCLUSIONS

The title of this report implied a goal to be reached at the end of this research endeavor; control of a nonlinear system. This goal was to be reached by solving the stated problem and accomplishing the objective (see p4). The measure of success depends on success in reaching the objective and problem solution. Therefore, objective and problem solution will be examined first.

The objective was to choose a gain which accomplished two sub-goals: 1. ensure stability in the linear system, and 2. stabilize the nonlinear system. In a system with a single unstable mode, current theory allows linear systems to be easily stabilized. Once the Poincare exponent is found a scalar constant gain can be used and the new system is stable (refer to equation (37) p13). However, with the nonlinear system also dependent on choice of gain for stability some gains that stabilize the linear system are rejected on the basis of not accomplishing the second sub-goal. Case 2 through 7 fall into this category. Before final judgment is pronounced on case 8, permit an examination of the problem and methodology of the solution.

The methodology for the problem solution relied on the Henon and Heiles problem to act as a test basis (5:73). The

Henon and Heiles potential function was used for its level of complexity yet ease of use. The linearized equations of motion, using Floquet theory, were developed into equations of variation. A controller was derived based on system characteristics: the controllability matrix, g(t). A first attempt, based on multiplying two Fourier series together to obtain the gain function k(t), was rejected due to high oscillations causing further instability in the nonlinear system. A second approach toward gain selection was more direct (refer to equation (47) p24) Inversion of g(t). and multiplying by a constant. This is the basis of cases 2-6. A refinement of this approach resulted in the last attempted gain selection, cases 7 and 8. As previously stated, each gain technique was used in the nonlinear system via a controller and checked for stability.

The methodology was straightforward and directly attacked the problem. A nonlinear system with a single unstable mode was controlled using a modal controller. A technique for gain selection was developed

$$k(t) = (\, w_i' - w_i)/\, g_i(t)\,) \qquad (47)$$

where k(t) is the gain, w_i = Poincare exponent / old modal exponent, w_i' = desired exponent / new modal exponent and $g_i(t)$ part of the controllability matrix. In this case the cutoff value of $g_i(t)$ was also recognized as significant. The cutoff value of $g_i(t)$ determined the on/off-time of the

controller. This ranged from 6.5 % in case 6 to 78.0 % in case 8. Case 8 was the only case that stabilized the nonlinear system.

Was the uniqueness of case 8 the ingredient that made it the only succesful case? Clearly the off-time is a marked increase over the other cases (refer to Fig 10. p36). The results of using this gain function, active for only 22.0 % of the time, reduces the deviations in Fig 8. by two orders of magnitude to Fig 11. (p32 & p37).

With this specific problem solved and the objective accomplished now let us go back to measuring the success of reaching the primary goal. Yes, a nonlinear system was controlled. But only in a limited manner. This report does not profess to having a methodology to control all other nonlinear sytems, yet it does make these statements:

1. It reaffirms the usefulness of the Henon and Heiles paper as a test bed for experimentation.

2. Floquet theory is a useful tool by which modal controllers can be implemented.

3. A new gain selection technique based on the system controllability matrix may provide future utility.

4. In this specific case, a relationship between the unstable Poincare exponent and the $g_i(t)$ cutoff value or on/off-time was uncovered.

5. The above relationship impacted heavily on the effectiveness of the controller and the ability to stabilize the nonlinear system.

From these conclusions several recommendations arise.

RECOMMENDATIONS

First, before any continued research is conducted a small investigation into whether the results of this report can be generalized for the area of modal control of nonlinear systems should be undertaken. Specifically, (1) whether the reference state, with its correspondind unstable mode, can be classified as a special case, and (2) whether the on/off-time characteristics of the controller depends on this classification.

Next, if the results were not of a special case and could possibly be generalized then a major decision must be made. Either use the Henon and Heiles paper or use an unstable nonlinear system with physical representations. Once this decision is made, I suggest, a trade-off study using the unstable Poincare exponent and the on/off-time ($g_i(t)$ cutoff value) as parameters.

Although research is being conducted involving two or more unstable modes , I believe investigation of the trade-off study should not be neglected. It might provide some insight or other help in conducting the multiple unstable mode research.

Finally, in a side note, I petition anyone in a position of authority with a technical problem to solicit engineering graduate schools for research. The masters thesis is an important part of the learning process in higher education and may provide some reciprocal benefits.

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Vita

Rosario Nici was born on June 22, 1956 in Bronx, New York. Raised in New York City, he graduated from Brooklyn Technical High School in June 1973. After 4 years at the USAF Academy, he accepted a Regular Commission in the United States Air Force. Subsequently, he entered Undergraduate Pilot Training at Reese AFB, in Lubbock, Texas. After graduation he was assigned to Kadena AB, Japan flying KC-135's. He was upgraded to Aircraft Commander 18 months prior to his leaving Okinawa, Japan in May 1983. Upon graduation from the Air Force Institute of Technology he will be an Instructor in the Department of Astronautics at the USAF Academy in Colorado Springs, Colorado.

He is married to the former Mary Jane Dopp, of Monument, Colorado and they have one child, Lindsay Marie.

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ABSTACT

A nonlinear system exhibited some relationship between system stability and controller off-time. An increase in the off-time of the controller from 6.5 % to 78.0 % resulted in the nonlinear system maintaining stability over 50 time periods.

A nonlinear system was controlled using feedback based on the unstable system parameters. The Henon and Heiles model was used to provide an unstable periodic orbit. The linearized equations of motion, using Floquet Theory, were developed into variational equations. A controller was derived based on a system characteristic: the controllability matrix. The controller gain was also calculated based on the controllabilty matrix.

Together with the nonlinear system the controller was integrated for many time periods using zero initial displacement. Stability was achieved for ~~approximately~~ about 50 time periods of the system around the zero displacement conditions. Other areas were investigated, along with several other controller gains, without any stability noted.

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